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Simple Numerical Model for Calculation of Entry Vehicle Trim Response

Thomas D. Burton*
Washington State University, Pullman, Wash.

THE influence of mass and configurational asymmetries on the dynamics of slender, spinning entry vehicles is an important aspect of vehicle design and performance assessment. High-altitude roll resonance and lower-altitude roll/trim effects influence vehicle loading and attendant impact miss. Descriptions of aerodynamic trim effects on re-entry vehicle (RV) angle of attack for vehicles with linear aerodynamics are well known.[1-4] Influence of nonlinear aerodynamics on resonant and nonresonant response have been considered by Murphy[5] and by Nayfeh and Saric.[6,7] Although these solutions are useful in characterizing angle-of-attack response, it is still generally necessary to numerically integrate the six-degree-of-freedom equations of motion in order to quantify the resulting trim-induced dispersion for a given re-entry system. This is often done using a Monte Carlo approach, necessitating significant amounts of computer time.

Presented in this Note is a simple trim response model which is numerically efficient and which can be incorporated into standard point-mass trajectory simulators for calculation of trim-induced dispersion and angle-of-attack/load behavior. This model is based on the fact that the coupling of trim and body fixed low-frequency (in the sense of Nelson[8]) oscillatory motion component dominates the trim-induced response. The high-frequency component/trim coupling is not significant, except in cases where sudden trim changes are encountered, e.g., due to rapid configurational change. By using standard asymptotic solutions to linear differential equations with time varying coefficients,[9] the two second-order equations in angle-of-attack α and sideslip β can be reduced to two first-order equations in which the high-frequency contribution is eliminated. This allows for a much coarser integration increment in calculating the trim-induced response.

In the present model, time variations in roll rate, asymmetries, and aerodynamic coefficients are taken into account. Longitudinal principal axis misalignments in pitch and yaw are also included, which was not done in the previously cited work.[1-7] In addition to the reduced equations of motion, results are obtained for the effect of roll acceleration and so-called density damping on the steady-state trim angle relations. The simplified model is shown to provide adequate accuracy in calculating the combined low-frequency/trim contribution to angle-of-attack.

The angle-of-attack behavior is formulated in body fixed coordinates as used by Nelson.[8] For small angles of attack, linear aerodynamics, and with inclusion of products of inertia and aerodynamic asymmetries for an otherwise symmetric vehicle, the equations of rotational motion may be written as follows:

$$\dot{\beta} = -r + p\alpha - (q_\infty S C_{L_\alpha}/mV)\beta \tag{1}$$

$$\dot{\alpha} = q - p\beta - (q_\infty S C_{L_\alpha}/mV)\alpha \tag{2}$$

$$\dot{q} = (1-A/B)pr + (q_\infty Sd/B)[C_{m_\alpha}\alpha + (d/2V)C_{m_q}q + C_{m_0}] - \epsilon_3(1-A/B)p^2 \tag{3}$$

$$\dot{r} = -(1-A/B)pq + (q_\infty Sd/B)[-C_{m_\alpha}\beta + (d/2V)C_{m_q}r + C_{n_0}] + \epsilon_2(1-A/B)p^2 \tag{4}$$

$$\dot{p} = (q_\infty Sd/A)C_l \tag{5}$$

Here, α and β are angles of attack and sideslip, respectively; p, q, and r are roll, pitch, and yaw rates, respectively; A and B are roll and pitch moments of inertia (pitch and yaw moments of inertia are assumed equal); vehicle mass and aerodynamic reference area and length are denoted by m, S, and d, respectively. Dynamic pressure is $q_\infty = \frac{1}{2}\rho V^2$, where ρ is atmospheric density and V is vehicle velocity. $C_{m\alpha}$, C_{mq}, $C_{L\alpha}$, and C_l are aerodynamic coefficients of static stability, dynamic stability, lift, and roll torque. C_{m0} and C_{n0} are aerodynamic trim-producing asymmetries in pitch and yaw, while ϵ_3 and ϵ_2 are pitch and yaw misalignments of the longitudinal principal axis.

Equations (1-4) can be combined into a single second-order equation in complex angle-of-attack $\xi = \beta + i\alpha$, written in the following form:

$$\ddot{\xi} + (2i\Delta\omega - 2\lambda_0)\dot{\xi} + (\omega^2 + 2i\lambda_1)\xi = T_0 \tag{6}$$

where

$$\Delta\omega = p(1-A/2B)$$

$$\lambda_0 = -(q_\infty S/mV)[C_{L_\alpha} - (md^2/2B)C_{m_q}]$$

$$\lambda_1 = (pq_\infty S/2mV)[(1-A/B)C_{L_\alpha} - (md^2/2B)C_{m_q}] + \dot{p}$$

$$\omega^2 = -(q_\infty Sd/B)C_{m_\alpha} - (1-A/B)p^2$$

$$T_0 = N_0 + iM_0$$

$$M_0 = (q_\infty SdC_{m_0}/B) + \epsilon_3(1-A/B)p^2$$

$$N_0 = -(q_\infty SdC_{n_0}/B) + \epsilon_2(1-A/B)p^2$$

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*Assistant Professor, Department of Mechanical Engineering. Member AIAA.

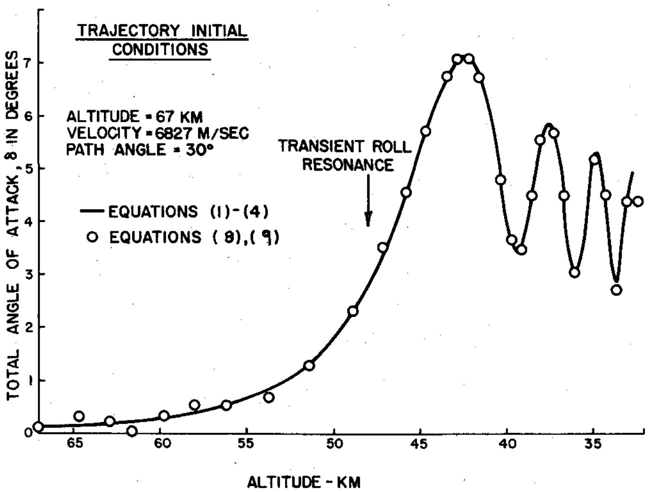

Fig. 1 Simulation comparison for $p=2\Pi$/s, $C_{m_\alpha}=-0.1$/rad, $C_{N_\alpha}=0$, $C_l=0$, $C_D=0.05$, $C_{m_0}=0.001745$, $C_{n_0}=0$, $\epsilon_2=0$, $\epsilon_3=0$, $S=0.13$ m^2, $d=1.9$ m, $m=115$ kg, $A=1.3$ kg-m^2, $B=22$ kg-m^2.

Equation (6) is a second-order linear differential equation with slowly varying coefficients, i.e., the change in coefficients in a cycle of oscillatory motion is considered small. Asymptotic solutions to such equations are well known. Noting that the damping terms λ_0 and λ_1 are small relative to the frequency terms $\Delta\omega$ and ω^2, the procedure given by Nayfeh[9] (to which reference should be made for details) enables an asymptotic first-order equation for the combined trim/low-frequency response to be written directly as

$$\dot{\xi}_L = [-i\omega_L + \lambda_L]\xi_L + [M_0 - iN_0]/\omega_H \tag{7}$$

where $\xi_L = \beta_L + i\alpha_L$ is the combined trim/low-frequency contribution to angle-of-attack, ω_L and λ_L are the low-frequency and low-frequency component damping rate, and ω_H is the high-frequency of oscillation,

$$\omega_L = \Delta\omega - \omega_0$$

$$\omega_H = \Delta\omega + \omega_0$$

$$\lambda_L = \lambda_0 + \Delta\lambda - (1/2\omega_0)(\dot{\omega}_0 + \dot{p}A/2B)$$

$$\omega_0 = [-(q_\infty SdC_{m_\alpha}/B) + (pA/2B)^2]^{1/2}$$

$$\Delta\lambda = \frac{q_\infty S}{mV}\left(\frac{pA}{4\omega_0 B}\right)\left[C_{L_\alpha} + \frac{md^2}{2B}C_{m_q}\right]$$

and λ_0 is as given previously. The frequency terms ω_L and ω_H are the usual body fixed low and high oscillation frequencies.[8] The low-frequency damping rate λ_L is composed of the usual aerodynamic and so-called density damping, $\dot{\omega}_0/2\omega_0$, and additionally, a small term due to steady roll acceleration $\dot{p}$, which has been determined previously by Platus.[10]

Separating Eq. (7) into real and imaginary parts, the individual equations for α_L and β_L are

$$\dot{\beta}_L = \omega_L\alpha_L + \lambda_L\beta_L + M_0/\omega_H \tag{8}$$

$$\dot{\alpha}_L = -\omega_L\beta_L + \lambda_L\alpha_L - N_0/\omega_H \tag{9}$$

These equations form the basis of the simplified numerical calculation of vehicle trim/low-frequency response. They are similar to results obtained using the method of multiple time scales by Nayfeh and Saric.[6,7] However, their analysis leads to separate first-order differential equations for amplitude and phase of angle-of-attack rather than separate equations for angle-of-attack components α_L, β_L. In addition, the influence of inertial asymmetries is included in the present model.

Equations (8) and (9) are especially useful in calculating high-altitude transient resonance response, since the integration time step is controlled by the low, rather than high, frequency. For application to roll trim dispersion analysis, the integration time step would be controlled by the roll rate or low frequency rather than the high-frequency. In either case, substantial reduction in overall integration time is possible.

Equations (8) and (9) can be solved for the steady-state trim angles ($\dot{\alpha}_L = \dot{\beta}_L = 0$), including the influence of roll acceleration and density damping,

$$\alpha_T = \frac{1}{\Delta}\left\{\frac{M_0\omega_L}{\omega_H} - \frac{N_0\lambda_L}{\omega_H}\right\}$$

$$\beta_T = \frac{1}{\Delta}\left\{\frac{N_0\omega_L}{\omega_H} + \frac{M_0\lambda_L}{\omega_H}\right\} \tag{10}$$

where $\Delta = -(\omega_L{}^2 + \omega_H{}^2)$. The contributions to roll acceleration and "density damping" are included through the λ_L term and appear because of coefficient variation in Eqs. (1-4).

Shown in Fig. 1 is a comparison of total angle-of-attack histories as obtained through integration of Eqs. (1-4) and through integration of Eqs. (8) and (9). The case shown is a high-altitude resonance traversal for a vehicle with constant roll rate and aerodynamic properties. Pertinent vehicle and trajectory characteristics are listed in the figure. The initial small oscillation using the present model resulted from inexact choice of initial conditions, i.e., a pure trim condition was not achieved. Agreement between the two simulations is good. In particular, the peak angle-of-attack attained and subsequent trim and low-frequency contributions are well modeled using the simplified approach.

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